Final Exam

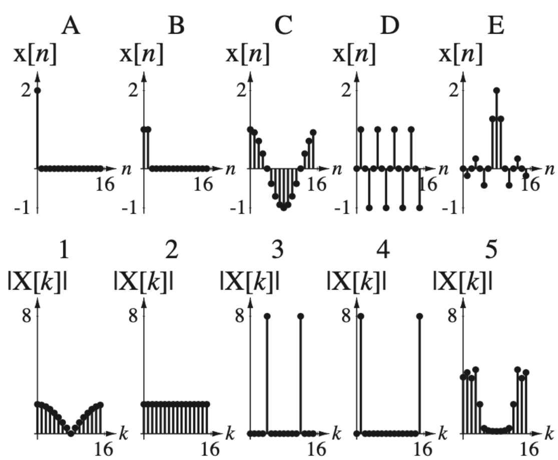
Signal and System, Fall 2021
School of BioMedical Convergence Engineering, PNU
Dec. 15. 15:00 - 23:59

I. REMARK

- This is an open book exam. You can use any materials if you want.
- There are a total of 100 points in the exam. Each problem specifies its point total.
- You must SHOW YOUR WORK to get full credit.
- If you just copy your classmate's answers or chat with anyone through any messenger, your total point would be 0.
- [MATLAB] implies that you need to use MATLAB. When you need to plot continuous-time signal x(t), please find the sampling rate f_s in the problem and plot the sampled signal $x[n] = x(t)|_{t/f_s}$. You need to display "time" on x-axis (not just discrete index). Also, when you need to plot $|(X(f)|$ (CTFT spectrum), use 'fft' and 'fftshift' functions and eq. 6.19 in the textbook to draw approximated one. You need to display 'frequency' on x-axis (not just discrete index).

II. PROBLEM SET

1) [5 points] In the figure below, match functions to their DFT magnitudes. Describe the reason of your answer in detail.

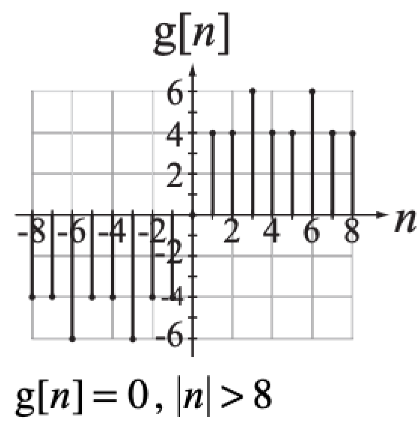


2) [10 points] [MATLAB] The signal $x(t)$ is given as $x(t) = 3\cos(20\pi t) - 2\sin(30\pi t)$ over a time range of $0 < t < 0.4s$. Graph the signal formed by sampling the function at the following sampling frequencies:
   a) $f_s = 120Hz$,
   b) $f_s = 60Hz$,
   c) $f_s = 30Hz$, and
   d) $f_s = 15Hz$.

   Based on what you observe, what can you say about how fast this signal should be sampled so that it could be reconstructed from the samples?

3) [10 points] Answer the following questions. You can use tables in the textbook.
   a) What is the CTFT of the function $h(t) = 2f_1\text{sinc}(2f_1t) - 2f_0\text{sinc}(2f_0t)$ where $f_1 > f_0$?
   b) If the impulse response function of one system is $h(t)$, what is the role of the system?
   c) [MATLAB] Plot $h(t)$ over $-5s < t < 5s$. $f_1 = 30Hz$ and $f_0 = 20Hz$. Use $100Hz$ for the sampling frequency f_s.
   d) [MATLAB] Plot $|H(f)|$ over $-50Hz < f < 50Hz$.

4) [10 points] The graphical definition of a function is given in the figure below.



   a) Graph $y[n] = g[n] * h[n]$ where $h[n] = (\delta[n] + \delta[n-1] + \delta[n-2]$.
   b) [MATLAB] Graph $g[n]$ and $h[n]$ using 'stem'. You can select any proper range of n when you plot the functions. Index n on x-axis must match the functions.
   c) [MATLAB] Compute $y[n]$ using 'conv' function. Graph $y[n]$ using 'stem'. You can select any proper range of n but you need to display all

nonzero points. Index n on x-axis must match the function $y[n]$.

5) [10 points] Answer the following questions.
   a) Prove that $Y(f) = H(f)X(f)$ if $y(t) = h(t) * x(t)$. Here, $Y(f)$, $H(f)$ and $X(f)$ are the CTFT of $y(t)$, $h(t)$ and $x(t)$, respectively. Use the definitions of CTFT and convolution.
   b) Prove that $Y[k] = X[k]e^{-j2\pi kn_0/N}$ if $y[n] = x[n - n_0]$. Here, $Y[k]$ and $X[k]$ are the DFT of $y[n]$ and $x[n]$

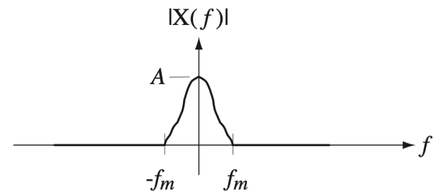
6) [15 points] The purpose of the task is making a song. Find the music (score) of the song below. For every scale, use a cosine or sine function. Use the table below describing the sinusoidal frequency of every scale. Assume that the time period for a quarter note is 0.5 sec. The sampling frequency $f_s = 1/T_s$ should be 44100 Hz.

옥타브 및 음계별 표준 주파수

옥타브 / 음계	1	2	3	4
C(도)	32.7032	65.4064	130.8128	261.6256
C#	34.6478	69.2957	138.5913	277.1826
D(레)	36.7081	73.4162	146.8324	293.6648
D#	38.8909	77.7817	155.5635	311.1270
E(미)	41.2034	82.4069	164.8138	329.6276
F(파)	43.6535	87.3071	174.6141	349.2282
F#	46.2493	92.4986	184.9972	369.9944
G(솔)	48.9994	97.9989	195.9977	391.9954
G#	51.9130	103.8262	207.6523	415.3047
A(라)	55.0000	110.0000	220.0000	440.0000
A#	58.2705	116.5409	233.0819	466.1638
B(시)	61.7354	123.4708	246.9417	493.8833

   a) [MATLAB] Use octave 4 for making the signal $x[n] = x(t)|_{t=nT_s}$ of the song. Plot $x(t)$ over time t. Also, plot $|X(f)|$ over frequency f. Use 'xlim([-1000 1000])' to see only the narrow range $-1kHz < f < 1kHz$. Listen the song x[n] using 'sound()'.
   b) [MATLAB] Make the song $y[n] = x[2n]$ through down-sampling. Plot $|Y(f)|$ over f. Also, use 'xlim([-1000 1000])' to see the range $-1kHz < f < 1kHz$. Listen the signal $y[n] = y(t)|_{t=nT_s}$ using same sampling frequency f_s. Describe any change of the sound in terms of song time and pitch. Explain reasons of the change in detail.
   c) [MATLAB] Using up-sampling and filtering, reconstruct the song $z[n] = z(t)|_{t=nT_s}$ from $y[n] = y(t)|_{t=nT_s}$ so that $z[n]$ is as close to $x[n]$ as possible. Use a sinc function for a filter $h[n] = h(t)|_{t=nT_s}$. Plot $|H(f)|$ and $|Z(f)|$ over f. Also, use 'xlim([-1000 1000])' to see the range $-1kHz < f < 1kHz$. Listen $z[n]$ using 'sound()'.

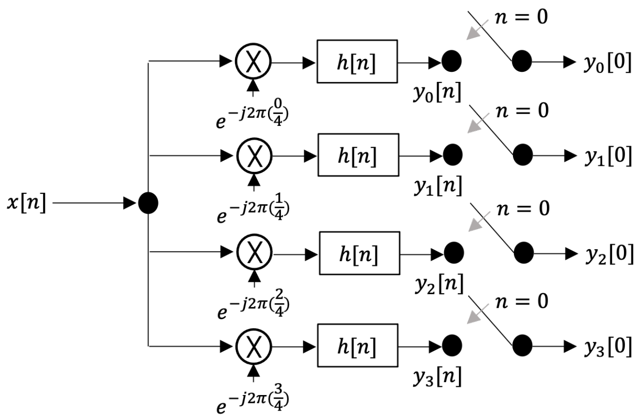
7) [10 points] $X(f)$ is the CTFT of $x(t)$. Suppose $A = 1$ and $f_m = 10Hz$. $x[n] = x(t)|_{t=n/f_s}$. The graph of $|X(f)|$ is given as



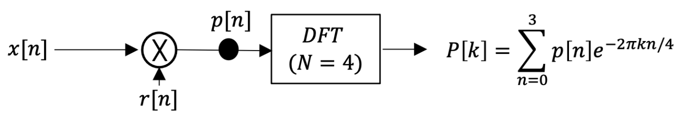
   a) Can $x(t)$ be time-limited signal? Demonstrate your answer in detail.
   b) Plot DTFT of $x[n]$ if the sampling frequency f_s is $30Hz$
   c) Suppose $y[n + 10k] = x[n]$ when $0 \leq n < 10$ for any integer k. Plot the approximate graph of $Y[k]$ where $Y[k]$ is DFT of $y[n]$. Use $N = 10$ for DFT.
   d) Suppose $y[n + 20k] = x[n]$ when $0 \leq n < 10$ and $y[n + 20k] = 0$ when $10 \leq n < 20$ for any k. Plot the approximate graph of $Y[k]$ where $Y[k]$ is DFT of $y[n]$. Use $N = 20$ for DFT.
   e) Repeat the questions b), c) and d) if $f_s = 16Hz$.

8) [15 points] Do as directed.
   a) [MATLAB] Download the file $'handel\_corrupted.mat'$ from Plato. Load the file though MATLAB. Then, you can see 'data' and 'Fs' where 'data' records the corrupted song and 'Fs' denotes the sampling frequency. Let $x[n] = x(t)|_{t=nT_s}$ be the corrupted data. Plot $x(t)$ over t. Also, plot spectrum $|X(f)|$ over f. Listen $x[n]$ using 'sound()'. Check that a beep sound disturbs one classical music. You can see strong peaks at $f = 120Hz$ and $f = -120Hz$ in the spectrum due to the beep.
   b) [MATLAB] Make a band-stop filter $h[n] = h(t)|_{t=nT_s}$ using a linear combination of one delta function and two sinc functions. Use $110Hz$ and $130Hz$ for cutoff frequencies of the filter so that the filter eliminates the strong beep. Plot spectrum $|H(f)|$ over f.
   c) [MATLAB] Conduct the filtering through $z(t) = h(t) * x(t)$. Plot spectrum $|Z(f)|$ over f. Listen $z(t)$ using 'sound'. You might clearly listen 'Hallelujah Chorus' from Handel's Messiah'.

9) [10 points] System 1 is shown in Fig.(a). In every channel, the filters are same. The impulse response of every filter is $h[n] = \alpha_0\delta[n] + \alpha_1\delta[n+1] + \alpha_2\delta[n+2]$. Every filter output is sampled at $n = 0$.



(a) System I



(b) System II

a) what is $y_k[0]$ in terms of $x[n]$ and $h[n]$
b) System II is shown in Fig. (b). If $P[k] = y_k[0]$, determine $r[n]$.